ON THE TOPOLOGIES OF SPACES OF LINEAR DYNAMICAL SYSTEMS COMMONLY EMPLOYED AS MODELS FOR ADAPTIVE AND ROBUST CONTROL DESIGN

Felipe M. Pait

Universidade de São Paulo – Laboratório de Automação e Controle
Av. Prof. Luciano Gualberto trav 3 158 São Paulo SP 05508-900 Brasil
pait@lac.usp.br

ABSTRACT

Robustness and performance of a closed-loop adaptive system may profit from the use of switching logic, whether or not that is required for stability. This logic may take the form of a supervisory switching scheme responsible for choosing, at each instant of time, one among several candidate control signals to close the feedback loop. Perhaps the most promising way to make this choice is with basis on observations of suitably defined error signals, corresponding to each candidate control. While that in itself does not preclude the use of a compact subset of a finite dimensional linear space as parameter set, we believe that the use of parallel hardware and software may make it advantageous to take as parameter space a finite set. However it is not clear that with a finite parameter set it is possible to stabilize as large classes of admissible process models as in conventional adaptive control. This note uses elementary topological concepts to show how this can in principle be done, by judicious choice of candidate controllers.

1. Logic Switching and Adaptive Control

Using a parameter estimation algorithm to tune a certainty-equivalence controller it is possible to adaptively stabilize fairly large classes of linear processes. When the parameter space contains points at which the design model (upon which certainty-equivalence systhesis is based) loses stabilizability, adaptive stabilization may require either the use of persistently exciting probing signals or some type of switching logic, such as cyclic switching [11] or hysteresis switching [8], together with more usual forms of dynamical systems [7].

Recent work [7, 9, 10] has indicated that robustness and performance of the closed-loop adaptive system may profit from the use of switching logic, whether or not that is required for stability. This logic may take the form of a supervisory switching scheme responsible for choosing, at each instant of time, one among several candidate control signals to close the feedback loop. Perhaps the most promising way to make this choice is with basis on observations of a suitably defined error signals, corresponding to each candidate control. While that in itself does not preclude the use of a compact subset of a finite dimensional linear space as parameter space, we believe that implementational issues may make it advantageous to take as parameter space a finite set. Evaluating the adequacy of n candidate controllers by observing n identification errors is a task which can be split among several processing units operating simultaneously, which makes it a natural candidate for the use of parallel hardware and software.

However it is not clear that such schemes are able to stabilize as large classes of admissible process models as is conventional adaptive control. This note uses elementary topological concepts (see, for instance, [1, 2]) to show how this can in principle be done, by judicious choice of candidate controllers. §2 presents some background material and terminology on adaptive control; more detailed material can be found in the references. §3 discusses process and design models commonly employed in adaptive and robust control, and §4 gives a topological construction of a space which is relevant for an adaptive-robust control synthesis.

2. Background

One way to think of a parameter adaptive control system is as the feedback interconnection of a process Σ_P and a parameterized controller $\Sigma_C(\widehat{p})$ whose parameter vector $\widehat{p}$ is adjusted by a tuner Σ_T [5]. From this point of view, a parameterized controller is a dynamical system depending on $\widehat{p}$, whose inputs are the process open-loop control u, the process output y, and possibly a reference input r; Σ_C's outputs invariably include not only a control signal u_C which in closed-loop serves as the feedback control u to the process, but also a suitably defined "tuning error" e_T which during adaptation drives Σ_T. In many instances Σ_C is "identifier-based" [6]. By this we mean that $\Sigma_C(\widehat{p})$

consists of two subsystems, one an "identifier" $\Sigma_I(\widehat{p})$ whose primary function is to generate an "identification error" e_I and the other an "internal regulator" $\Sigma_R(\widehat{p})$ whose output is u_C. In such cases e_T is often chosen to be the same as e_I.

3. Process and Design Models

The starting point for the development of an identifier-based parameterized controller for an adaptive system is usually the specification of a class $\mathcal{C}_P$ of admissible process models Σ_P which the adaptive algorithm is supposed to control. In this paper we consider classes of process models consisting of n_u-input, n_y-output, controllable, observable linear systems, seen as elements of a space endowed with the graph (or gap) topology on their transfer function matrices[1]. Given $\mathcal{C}_P$, the first step in the design of the parameterized controller is typically to select a parameter space $\mathcal{P} = \mathbb{R}^{n_p}$ and a *design model* $\Sigma_D(p)$ upon which its design is to be based. $\Sigma_D(p)$ is chosen to have the same control input and measured output value spaces as the process models in $\mathcal{C}_P$, and also so that the transfer matrix of each such process model is "close" in some sense to the transfer matrix of $\Sigma_D(p)$ for some $p \in \mathcal{P}$. The reduced transfer matrices of $\Sigma_D(p)$ as p ranges over $\mathcal{P}$ are the transfer matrices which $\Sigma_D(p)$ *matches.*

We can partition $\mathcal{P}$ in two subsets: the singular region $\mathcal{S}$ which contains all points $p \in \mathcal{P}$ at which $\Sigma_D(p)$ is not stabilizable and its complement $\bar{\mathcal{S}} = \mathcal{P} - \mathcal{S}$. Notice that $\bar{\mathcal{S}}$ contains precisely those points at which addition and multiplication of $\Sigma_D(p)$'s transfer matrix are not continuous operations under the graph topology! Identification theory makes a convexity requirement on the set of admissible parameter values. However, defining $\bar{\mathcal{S}}$ as a convex set, or even the union of finite sets, is a difficult problem and often leads to severe restrictions on the class of matchable (or approximately matchable) process models. Hence the use of tuners based on parameter estimation forces us to consider tuning on all of $\mathcal{P}$, not only on $\bar{\mathcal{S}}$, and thus to deal with the problem of loss of stabilizability of the design model. Certainty equivalence controllers can be defined on the whole $\bar{\mathcal{S}}$, but certainly not on $\mathcal{S}$ due to the stabilizability question. Since the tuners we are concerned with are of a different type, we need not worry about the convexity requirement. Therefore we take $\bar{\mathcal{S}}$ as effective parameter set.

[1] Recall that the graph topology is the weakest (coarsest) topology in which feedback stability is a robust property, and that it is the topology induced by either the gap metric or the graph metric. The operations of transfer matrix addition and multiplication are continuous under these metrics at all points except those where unstable pole-zero cancelations occur. For discussions on these topologies and the asscociated metrics, see e.g. [3, 4, 12].

We wish to investigate whether it is possible to find an "interesting" class $\mathcal{C}_P$, a number n, and a set of candidate linear time-invariant controllers $\{\Sigma_C(p_1), \Sigma_C(p_1), \cdots, \Sigma_C(p_n)\}$ with the property: for each process $\Sigma_P \in \mathcal{C}_P$ there exists an integer i such that the feedback interconnection of Σ_P with $\Sigma_C(p_i)$ is stable in the system-theoretic sense. That is to say, $\Sigma_C(p_i)$ robustly stabilizes a set $\mathcal{C}_{P_i} \subset \mathcal{C}_P$, and the $\mathcal{C}_{P_i}$s are such that $\cup_i \mathcal{C}_{P_i} = \mathcal{C}_P$[2]. First we make the following considerations:

1. $\mathcal{P}$ is a bounded set.
2. Since $\mathcal{P} \subset \mathbb{R}^{n_p}$, which is finite-dimensional, $\mathcal{P}$ is compact.
3. $\bar{\mathcal{S}} \subset \mathcal{P}$ thus $\bar{\mathcal{S}}$ is compact as well.
4. Define $\mathcal{X} = \{\Sigma_D(p)$ as p ranges over $\bar{\mathcal{S}}\}$.
5. $\bar{\mathcal{S}}$ contains only points at which the gap topology is continuous.
6. Thus the norm associated with the gap metric is continuous in $\mathcal{X}$, and $\mathcal{X}$ is compact as well.
7. Meanwhile, $\mathcal{C}_P$ is in general *not* compact, because we are interested in controlling plants with unmodeled dynamics, which can be infinite-dimensional.
8. Thus a finite cover for $\mathcal{C}_P$ cannot be constructed as a subset of an arbitrary open cover.

The above highlights a basic difference between the classes of models used in robust and in adaptive control: while the former deals with unstructured, non-compact sets of admissible processes around a nominal model, the latter is concerned with parameterized classes of design models, which form a compact set as soon as the parameters are taken to be bounded. The following section shows how to define a suitable candidate for $\mathcal{C}_P$ in the context of a supervisory ("higher-level") switching control scheme, and how to construct a finite cover for it.

4. A Topological Construction of $\mathcal{C}_P$

Let $\mathcal{X}$ be a compact set in a metric space $\mathcal{M}$. For each point $x \in \mathcal{X}$, take an open ball $\mathcal{B}(x, \delta(x))$ centered in x with radius $\delta(x)$, where $\delta(x)$ is some continuous function of x. Then define the set $\mathcal{Y}$ as the union of all such balls. In general, $\mathcal{Y}$'s closure is not compact unless $\mathcal{M}$ is finite-dimensional. Thus a finite cover for $\mathcal{Y}$ cannot be constructed as a subset of an arbitrary open cover. We are interested in the following

Problem: Construct a finite cover for $\mathcal{Y}$.

[2] The question of how to select in real time a feedback control signal amongst the candidates is discussed in [10].

A construction is given below. Let $\{\mathcal{B}_i(x_i, \delta_i)\}, i = 1, 2, \cdots, n$, be a finite open cover of $\mathcal{X}$, formed by balls of radius δ_i centered around x_i. Assume without loss of generality that

$$\nexists j : \mathcal{B}_j \subset \cup_{i \neq j} \mathcal{B}_i$$

(otherwise, simply remove $\mathcal{B}_j$ from the set $\{\mathcal{B}_i\}$, and then repeat the operation successively until the assumption holds). Such a cover exists because $\mathcal{X}$ is compact. That means

$$\forall x \in \mathcal{X}, \exists i \in [1, n] : ||x - x_i|| < \delta_i.$$

For each point $x \in \mathcal{X}$ let $\mathcal{I} \subset [1, n]$ be the set of indices i such that $x \in \mathcal{B}_i(x_i, \delta_i)$. Let $\bar{\delta}(x) = \min_{i \in \mathcal{I}} \delta_i$, with $i(x)$ the corresponding value of the subscript, and define $\bar{\delta}_i = \sup_{x \in \mathcal{B}_i} \delta(x)$. Clearly $\bar{\delta}_i$ is finite, because $\delta(x)$ is continuous and $\mathcal{B}_i$ is bounded. Around each x_i pick a ball $\mathcal{V}_i$, of radius $r(x_i) = \bar{\delta}_i + \delta_i$. Now let $y \in \mathcal{Y}$. By definition,

$$\exists x : ||y - x|| < \delta(x).$$

With $i(x)$ as defined above,

$$||y - x_i|| \leq ||y - x|| + ||x - x_i|| < \delta(x) + \delta_i \leq r(x_i),$$

so $y \in \mathcal{V}_i$. Therefore $\forall y \in \mathcal{Y}, y \in \cup_i \mathcal{V}_i$, and $\{\mathcal{V}_i\}$ is a finite cover of $\mathcal{Y}$, as we were looking for. Moreover, it is not difficult to see that $\{\mathcal{V}_i\}$ is the "tightest" cover of $\mathcal{Y}$ that can be constructed with basis on the cover $\{\mathcal{B}_i\}$ of $\mathcal{X}$.

5. Conclusions

We submit that the set $\mathcal{Y}$, as constructed in §3, is a good candidate for the role of class of admissible process models ($\mathcal{C}_P$). The uncertainty contained in the assumption $\Sigma_P \in \mathcal{Y}$ is of a combined structured and unstructured type; structured uncertainties have traditionally been the object of study of adaptive control, while unstructured uncertainties are dealt with by robust control. The construction given here serves to delineate what can be achieved using an identifier-based supervisory control system capable of switching a sequence of robust controllers, each a member of a finite set, into feedback with a process subject to both types of uncertainty.

Acknowledgements

The author thanks A. S. Morse, Paulo Sérgio Pereira da Silva, Oswaldo L. V. Costa, and Fuad Kassab Jr. for helpful discussions concerning this work.

This research was supported by FAPESP – State of São Paulo Research Council, under grant 91/0508-3, by CNPq – Brazilian Research Council – grant 520961/93-5, and by the Rosa G. and Henrique S. Pait Foundation for the Advancement of the Arts and Sciences.

6. References

[1] J. Dieudonné. *Foundations of Modern Analysis.* Academic Press, New York, 1969.

[2] J. Dieudonné. *Treatise on Analysis.* Academic Press, New York, 1970.

[3] A. K. El-Sakkary. The gap metric: Robustness of stabilization of feedback systems. *IEEE Trans. Automatic Control*, 30(3):240–247, Mar. 1985.

[4] T. T. Georgiou. On the computation of the graph metric. *Systems & Control Letters*, 11:253–257, 1988.

[5] A. S. Morse. Towards a unified theory of parameter adaptive control – Tunability. *IEEE Trans. Automatic Control*, 35:1002–1012, Sept. 1990.

[6] A. S. Morse. Towards a unified theory of parameter adaptive control – Part 2: Certainty equivalence and implicit tuning. *IEEE Trans. Automatic Control*, 37(1):15–29, 1992.

[7] A. S. Morse. Supervisory control of finite families of linear set-point controllers. In *Proceedings of the IEEE Conference on Decision and Control*, pages 1955–1960, Dec. 1993.

[8] A. S. Morse, D. Q. Mayne, and G. C. Goodwin. Applications of hysteresis switching in parameter adaptive control. *IEEE Trans. Automatic Control*, 37(9):1343–1354, Sept. 1992.

[9] K. S. Narendra and J. Balakrishnan. Performance improvement in adaptive control systems using multiple models and switching. In *Proc. Seventh Yale Workshop on Adaptive and Learning Systems*, pages 27–33, New Haven, Connecticut, May 1992.

[10] F. M. Pait and F. Kassab Jr. Parallel algorithms for adaptive control: Robust stability. In *IFAC Symposium on Robust Control Design*, Rio de Janeiro, Brazil, Sept. 1994.

[11] F. M. Pait and A. S. Morse. A cyclic switching strategy for parameter adaptive control. *IEEE Trans. Automatic Control*, 39(6):1172–1183, June 1994.

[12] M. Vidyasagar. The graph metric for unstable plants and robustness estimates for feedback stability. *IEEE Trans. Automatic Control*, 29(5):403–418, May 1984.