On the Design of Direct Adaptive Controllers*

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Abstract

Direct adaptive control systems without the familiar reference models is considered. A framework for design using quadratic cost functions is presented, and corresponding error equations are derived using ideas from linear-quadratic optimal control.

Key Words: Adaptive Control; Direct Control; Linear-Quadratic Optimal Control.

1 Direct Adaptive Control

The first step in the design of a *direct* adaptive control system is to decide on the underlying control design methodology. The adjustable control parameters, the shape of the error equation, the ever-present adaptive observer, all follow from this initial choice. This contrasts with indirect adaptive control systems. Indirect controllers typically comprise a parameterized observer which generates an identification error; a certainty-equivalence feedback regulator; and a tuner or adaptive law. These components can be designed in a modular fashion, more or less independently, provided each possesses some properties which are indeed satisfied by typical control and estimation algorithms.

The overwhelming majority of the direct adaptive control literature uses reference models as the design paradigm. This is because the control error between a plant's output and that of a suitably defined reference model can be expressed in a convenient form in which the control parameters appear linearly — provided, of course, that a number of restrictive hypothesis are satisfied. Another class of direct controllers are non-identifier based universal controllers (see [1] for a current presentation of direct, indirect, and non-identifier based adaptive control).

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However, reference models are just one possibility in indirect adaptive control, and they are used sparingly outside adaptive control if at all. In this paper we explore the feasibility of another design technique: linear-quadratic optimal control. Design using a quadratic objective is perhaps the most transparent and best understood paradigm that can be applied to detectable, stabilizable linear dynamical systems in general. (Interestingly, universal adaptive controllers often employ quadratic cost functions.) In the sequel we show that a parameterized controller can be constructed using tools from linear-quadratic optimal control. Suitable tuners to complete the adaptive feedback loop are the subject of a forthcoming paper [4].

2 Framework

We wish to control a plant with measured output $y \in \mathbb{R}^{n_y}$ and control input $u \in \mathbb{R}^{n_u}$ using an n-dimensional identifier-based direct adaptive controller of the form

$$\dot{x} = A_I x + B_I u + D_I y \tag{1}$$

$$u = Fx. \tag{2}$$

The "adaptive observer" or identifier given by (1) can be constructed as spelled out in [2, 3].[1] The plant to be controlled must satisfy the

Gardening Hypothesis: There exist an $n_y \times n$ matrix C_* and an $n_y \times n_y$ singular, strictly lower triangular matrix G_* such that the pair $(A_I + D_I(I - G_*)^{-1}C_*, B_I)$ is stabilizable and moreover

$$\tilde{y} = C_* x - (I - G_*)y \tag{3}$$

is small in some well-defined sense (for instance, $\tilde{y} \in \mathcal{L}^2$ on some time interval; or $\tilde{y}$ has bounded power; or yet $\tilde{y} = \Delta(s)u(s)$ with the stable transfer function Δ subject to some operator norm bound). ♣♣

The point of this hypothesis is that if the construction in [3] is followed there must exist an $n_D \times n$ matrix $E_I(C_*, G_*)$ such that $\widehat{x} = E_I x$ is a nonminimal asymptotic observer for the n_D-dimensional stabilizable, detectable "nominal plant" or design model

$$\dot{x}_D = (A_D + D_I(I - G_*)^{-1}C_D)x_D + B_D u \tag{4}$$

$$y = (I - G_*)^{-1}C_D x_D. \tag{5}$$

Thus if $u = \bar{F}x_D$ is a state feedback that stabilizes (4), the separation principle tells us to use as dynamic output feedback

$$u = \bar{F}\widehat{x} = \bar{F}E_I x.$$

That is to say, in order to stabilize the process (4), (5) we might use controller (1), (2) with $F = \bar{F}E_I$. Of course the hypothesis is somewhat restrictive; it implies that the real plant's input-output behavior is close to that of the nominal model.

[1] If the plant is SISO ($n_y = n_u = 1$), then one could pick the matrix triple (A_I, B_I, D_I) as follows: choose $n/2 > 0$ and a controllable pair $(\bar{A}_{n/2\times n/2}, \bar{b}_{n/2\times 1})$ with $\bar{A}$ stable. Then define $A_I = \begin{bmatrix} \bar{A} & 0 \\ 0 & \bar{A} \end{bmatrix}$, $B_I = \begin{bmatrix} 0 \\ \bar{b} \end{bmatrix}$, and $D_I = \begin{bmatrix} \bar{b} \\ 0 \end{bmatrix}$. In this case $G_* = 0$.

We may substitute the value of y given by (3) into (1):

$$\dot{x} = (A_I + D_I(I - G^*)^{-1}C^*)x + B_I u - D_I(I - G^*)^{-1}\tilde{y}.$$

If we can choose $F(t)$ in a way that keeps the trajectories of the system above bounded, then u and y must both remain bounded; detectability and stabilizability of the plant then guarantee that *all* signals in the overall system remain bounded. Naturally we are interested in the case where neither C_* nor G_* are known! From now on we shall write $A = (A_I + D_I(I - G_*)^{-1}C_*)$, $B = B_I$, and $w = -D_I(I - G_*)^{-1}\tilde{y}$, and concern ourselves with the problem of adaptively stabilizing a process

$$\dot{x} = Ax + Bu + w \tag{6}$$

using state feedback control (2), where A, B, and w are unknown, A and B are fixed, and w is "small."

3 Error Equation

Choose constant, symmetric, nonsingular matrices Q and R and define, for each matrix $\widehat{P}$, a tuning error

$$e_T = \dot{x}^T \widehat{P} x + x^T \widehat{P} \dot{x} + x^T Q x + u^T R u. \tag{7}$$

Notice that x's derivative can be obtained from (1) so e_T is a computable quantity. Motivation for the definition above is as follows. Stabilizability of (A, B) in (6) assures that there exists a positive-definite solution to the algebraic Riccati equation

$$A^T P + PA - PBR^{-2}B^T P + Q^2 = 0$$

for any choice of positive-definite matrices Q^2 and R^2. Thus

$$\begin{aligned}
\frac{d}{dt}x^T P x &= x^T PAx + x^T PBu + x^T Pw + x^T A^T Px + u^T B^T Px + w^T Px \\
&= -x^T Q^2 x + x^T PBR^{-2}B^T Px + x^T PBFx + x^T F^T B^T Px + 2x^T Pw \\
&= -x^T Q^2 x - u^T R^2 u + x^T (F + R^{-2}B^T P)^T R^2 (F + R^{-2}B^T P)x + 2x^T Pw.
\end{aligned}$$

In order to minimize e_T we may thus resort to the following identity:

$$e_T = \dot{x}^T(\widehat{P} - P)x + x^T(\widehat{P} - P)\dot{x} + |R(F - F_*)x|^2 + 2x^T Pw, \tag{8}$$

where F_* denotes the (unknown) feedback gain matrix $-R^{-2}B^T P$.

If we can adjust the direct adaptive control parameter F and the state cost matrix estimate $\widehat{P}$ in a manner that keeps e_T small in some sense without allowing $\widehat{P}$ to blow up, then we have achieved a form of direct stabilization *without a reference model.* How one might deal with this problem, at least in the single-input case (F is an $n \times 1$ vector), is the subject of a forthcoming paper [4].

4 Concluding Remarks

If we abstract for a moment the terms in $\widehat{P}-P$ and w, (8) looks very much like a traditional parameter estimation problem. It would then be amenable to treatment by a least-squares algorithm, except that in the present case the error is known in magnitude only. The need arises for tuners with capabilities comparable to the traditional least-squares or gradient-type, which can function with information on the magnitude of the error only. With the help of such a tuner one can close a direct adaptive feedback loop without making the usual restrictive assumptions of minimum phase, known relative degree, etc.

References

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